Southwestern University

Guide for Writing in Mathematics

About Writing in Mathematics

Writing well in mathematics takes practice, careful composition, revision, and proofreading, as well as attention to detail and to audience. In the natural sciences in general, writing should be clear and objective rather than creative. Writing in mathematics often involves fluidly blending precise description with proofs, algorithms, calculations, code,

equations, tables, graphs, and other modes of output. Math writing is precise and concise.

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics outlines a set of skills that all undergraduates should demonstrate. According to their 2004 Curriculum Guide, students in mathematics should do the following:

"

- State problems carefully.
- Articulate [your] assumptions.
- Appreciate the value of precise definition.
- Reason logically to conclusions.
- Interpret results intelligently.
- Read mathematics with understanding.
- Communicate mathematical ideas with clarity and coherence through writing and speaking.

"

When writing for your math courses, you will not only clarify and ensure your own understanding, you will exhibit that understanding in a way that enhances your audience's knowledge and insight.

By Dr. Therese Shelton

About this guide

This guide contains the following sections:

- About Writing in Mathematics
- Common Types of Writing in Mathematics
- Evidence in Mathematics
- Conventions of Writing in Mathematics
- Citations & Formatting
- Further Resources

The Disciplinary Writing Guides are designed to provide an introduction to the conventions, or rules, of writing in different subjects. These guides have been designed by Southwestern professors to help you understand what will be expected of you in your classes.

Types of Writing in Mathematics

As an undergraduate mathematics student, you will probably need to write short answers, proofs, papers of varying lengths, and/or a more extensive research paper with an abstract. You might also need to compose a summary poster or a ten-minute oral presentation supported by slides.

Although these assignments vary in length and organization, what they share in common is that all of them ask you to demonstrate the **flow of logic**, **evidence**, and **justification** for a calculation, conjecture, theorem, classification, model, algorithm, or computer program.

Short Answer Calculation or Derivation

A short answer calculation or derivation requires brief justification, often in phrases. This may consist of a named result that is used (e.g., "l'Hopital's Rule," "definition of linear independence") or of a well-labeled diagram that implicitly defines a variable.

Proof

A proof generally consists of a concise statement of the result to be proved, often identified as "Theorem" (or "Lemma," etc.), followed on a new line by "Proof:" as a way to set up a well-reasoned argument. A proof often begins with a statement of what is to be shown.

Short Paper

A short paper may involve an exposition, results to be proven, external program documentation, or classification (e.g., types of differential equations, search methods, special functions, or directed graphs.) Explanations may be aided by including a specific clarifying example intermixed with the topic in general.

Computer Code

Computer code may be written either as short answer calculations, exhaustive enumerations, simulations, implementation of algorithms, or mathematical models. Fundamental programming guidelines must still be followed in mathematics; meaningful variable names and program documentation are essential for clarity and communication. Comments within the code include a brief description of the problem, an overview of the solution, and explanations throughout the code addressing the logic and approach of the implementation.

Abstract

An abstract is often required for an oral presentation (particularly when submitting to a conference) or for a longer paper. Abstracts allow readers to decide if they want to read a full paper or to attend a conference presentation. The abstract provides a brief overview, hints at the paper or presentation's main results, and (if appropriate) includes a context that indicates the importance of the work. Because abstracts are often written before the work has been completed, wording is often intentionally vague and broad. When writing your abstract, pay special attention to the instructions: some venues require a minimum and/or maximum word count.

Longer Paper

A longer paper is often subdivided into sections, which will vary depending on the topic of the paper. For example, a paper about a mathematical model might include five sections. (1) The introduction, where the context and/or necessary background are given. This should include an outline of the situation or problem that will be dealt with. (2) The primary model, possibly preceded by some preliminary models, which is outlined with supporting equations, tables, graphs, etc. (3) A critique of the model(s) that discusses both strengths and weaknesses, which might include complexity vs. accuracy. (For instance, a fourth degree polynomial fit might only marginally perform better than a quadratic.) One algorithm might be preferred for its simplicity but have a poor average completion time or a narrow interval of convergence. (4) An exploration of future areas of work, which is common in the final portion of a paper. (5) A references and acknowledgements section, which allows readers to learn more about the work.

In an oral presentation, you should prepare slides that organize key information in

Oral Presentation

bulleted lists rather than complete sentences. Although the number and order of slides will be tailored to the content of your presentation, slideshows generally follow the layout of a longer paper. You can think of the first slide as your **title** page. Usually, the first slide includes the author, their institutional affiliation (Southwestern University), the title of the venue (conference title), and the date of the presentation (at least month and year). Other options include the institutional logo, the conference logo, and the course or experience that produced the work. The abstract might be included, though it is not generally read aloud. The next few slides provide **introduction or background.** These are followed by several slides of the **results**, with supporting output, graphs, etc. The presentation continues with **concluding remarks**, which might include strengths or weaknesses, or a preview of future work. The last few slides are generally devoted to acknowledgements, references, and contact information. Acknowledgements are generally included on one of the last few slides. The software or programming environment may be listed here. A slide of references should give complete citations. A final slide may provide contact information, generally an email address. As you prepare the slides for your oral

Poster

A poster provides an overview of your work and should serve as a stand-alone product as well as a "prop" for an oral presentation. There are a number of very effective PowerPoint templates that you can use to make posters. These can be modified fairly easily, although boxes and titles are often in layers that can be somewhat troublesome to control. **Time, patience, and frequent file saving are important** when creating your poster. Most templates are designed for a

presentation, a good guideline is to have a maximum of six words per line and six lines per slide (except for the slides providing references).

Poster (continued)

particular size, which is often an essential consideration: conferences generally specify the dimensions of posters. A poster is generally composed in much the same manner as a set of slides. A poster, set of slides, or paper will often include the course or experience which produced the work, for instance: "In partial fulfillment of the Capstone in Computer Science."

Evidence in Mathematics

In mathematics writing, evidence must be substantiated, and evidence must be clear. For instance, a regression model should include the equation, the value of the correlation, or the coefficient of determination. A hypothesis test should include the type of test being performed, the sample size(s), the p-value, the level of significance, and the conclusion, both in statistical jargon and in the context of the situation. Often multiple fits to a set of data will be compared by an error analysis, such as sum of standardized residuals.

Below are a few considerations to keep in mind as you work on your writing in mathematics:

Level of Abstraction

In math writing, abstraction and generalization are often the focus of the content. For a mathematical model of specific data, the process and rationale are described so that they may be applied to another set of data. Therefore, examples are often generalized to a conjecture or proven statement. Code in a specific language may be generalized to an algorithm expressed in pseudocode.

Level of Detail

Both students and professionals sometimes feel a bit cheated if they worked hard to produce calculations, code, or a proof, but the detail exceeds what seems prudent to include in the main body of the paper. On the other hand, if there is interesting history or context, the author should ask "Where is the math?" as a test of whether the material is an aside. For instance, in a simulation of a tennis match, how much about the lives of the players or the equipment contributes to an understanding of the simulation? In these cases, it may be appropriate to locate the detail in an appendix. Details about tennis playing styles that justify the assignment of probability values or the choice of a distribution would be in the main document.

Data and Manipulation

Stating a sample of the data within the body of a paper is generally sufficient to aid in the explanation of processing the data. Your focus should be on the methodology used rather than laborious spreadsheet computations.

Sources

The use of outside sources in mathematics writing varies depending on the assignment. Outside sources might be appropriately forbidden in an assignment designed to assess a student's original and untainted thoughts and style of reasoning. Sources may be optional, or they may only be allowed to help spur ideas, particularly in expository writing. Or sources may be required, especially if the purpose is to assess reading comprehension or to perform a literary review. Sources are essential when conducting a survey of current open problems, providing a historical perspective, or outlining recent advances, solution techniques, or well-established theory or practice.

Conventions of Writing in Mathematics

The writer in mathematics must have both something to say and a clear writing style so the result is worth the time and attention required of the reader. Proper grammar, punctuation, and sentence structure are essential and, of course, the mathematics must be correct!

- Having a target audience in mind—peers, experts in the field, or a lay audience—guides the balance between description and detail. In an introductory computer science course, for example, it may be appropriate to explain the concept of iteration, while in an upper level course this would normally be assumed. In Calculus I, a student should show the steps involved with a change of variable, but in a later course, "The following results from the substitution u=lnx," with the detail omitted, would be appropriate.
- Writing in passive voice is common in mathematics since
 the focus is almost always on the work itself. Using "I" in
 a reflective paper is generally appropriate, and for other
 types of writing, "we" may be used occasionally.
- Writing in mathematics should be careful of tense. When
 describing facts, use present tense (facts are true).
 When describing experiments or methods, use past tense
 (experiments were conducted). Hamilton College
 Writing Center's "Writing for Science" resources
 provide helpful models.
- There are a few words that are commonly confused.
 "Amount" is used for values that can vary continuously, such as volume, weight, temperature, or biomass;
 "number" is used for values that are discrete, such as an actual count.
- Avoid contractions, slang, text-speak, or lofty words.
 Use the actual noun rather than ambiguous pronouns such as "they," "this," or "it."
- Definitions that are required within a paper might be indicated with "Definition," or the word or phrase being defined might be bolded or italicized without the overt label.
- Whether a single problem or a 20-page paper, writing in mathematics should demonstrate a clear flow of logic.
 Some transitional words commonly used in math writing include "moreover," "furthermore," "therefore," "thus,"

- and "hence." Logical reasoning must be apparent in the clarity and organization of the writing. As Steven Krantz notes in *A Primer of Mathematical Writing*, "The sentences that link the mathematics are usually most effective when they are simple, declarative, sentences" (14).
- Generally, an abstract is single-spaced, and a longer paper is double-spaced. Most multi-page writing benefits from a header and footer. The header often includes the author's last name and the title; the footer contains a page number. The first page is not numbered, especially if it is a title page. When multiple drafts are required for instruction, including the date or "Draft 2" on the title page or in the header is helpful.
- A technical paper or a proof often contains equations, tables, graphs, and other figures; a numbering convention is often needed. In general, numbering within a category is consecutive. For instance, one might have "Table 1," followed by "Figure 1," and then "Table 2." For a long paper in which sections are numbered, labels might be "Table 2.1," "Table 2.2," etc. Include a number for an equation that is referred to in a discussion. Label every table or figure; numbering improves clarity when there are many or similar graphs or tables. A table or figure often has a caption, such as "Figure 3: Sample Data of Measles Cases in China."
- Mixing notation with words is generally necessary, but a sentence should not begin with a symbol. For instance, Krantz suggests rewriting "Let f be a function. f is said to be semicontinuous..." as "A function f is said to be semicontinuous if..." (24).
- Aim for a balance between expressions or short equations embedded in text and those displayed and indented on a line by themselves.
- Do not recycle symbols in close proximity; do not use x for two different meanings. Conversely, do not use two different symbols for the same entity.

Citations and Formatting

Crediting Sources

There is no standard citation style that applies to all of math and computer sciences writing. You should consult your instructor or, if appropriate, conference or journal guidelines. Consistency in using a citation style within a document is important. Although style guides do not generally require URLs, an instructor may require them.

If any source is used, the writer is bound by the general rules of academic integrity, an ethic that must be developed and learned. Results must be reported accurately; changing data is considered falsifying and is an example of dishonesty. Credit must be given where credit is due. In mathematics writing, paraphrasing is often better than direct quotes. Often the author will begin a passage with, "According to Futamura and Anthony in [3]," and then will paraphrase, rather than quote, the relevant ideas that are used.

You need not credit common knowledge, such as the primary trigonometric identity $\sin^2 x + \cos^2 x = 1$. This can be tricky when you're writing about a subject that is new to you. If in doubt, consult with your instructor or go ahead and include the citation.

Data, figures, graphs, algorithms, or special functions or equations should generally be cited if not generated by the author or already included in the author's previous work. If you create a graph from someone else's data, for instance, the graph would not be cited but the data would. Any image created by someone else requires citation.

Often in the course of research, **sources that were investigated may prove to be extraneous. These sources should not be included among "References,"** but may be listed as "Further Reading." When you've conducted an extensive literature review, an "Annotated Bibliography," including both the reference and a brief description of content, can be a valuable inclusion to your paper.

Acknowledgements may include the supervising instructor, other instructors who were consulted, a reference librarian, the Writing Center, etc. **Permission** must be obtained to include details of a private organization or sensitive information of an individual.

Graphs and Figures

Tips for creating clear graphs

Using a common numerical scale on multiple, related graphs appears more professional and enables your readers to easily draw comparisons between the graphs. For example, if you created graphs of the populations of different states within the USA over a

ten-year period, formatting the y-axis of each with the same interval of values makes the relative sizes immediately apparent. Care must be taken, however, not to let one or more data sets overwhelm the others. In a graph illustrating accidental causes of death, for example, the number of deaths by sky diving would be a mere blip compared to the number in an auto accident. In this situation, you might make separate graphs or combine several small causes together, perhaps in an "other" category. Alternatively, rates should be shown rather than raw numbers, as in the rate of a condition, equal to the number of cases divided by the population, over time per country. Within a single data set, a logarithmic scale can aid in comparing values of different magnitudes, such as CEO salaries, which might be one dollar per year (Michael Dell) or five million.

Intentionally formatting the axes scales is generally helpful. For instance, if all of the y-axis values are in units of hundred thousands, then a scale of 1.1, 1.2, etc. is often preferred to 110,000; 120,000; etc. In this case, an indication that the units are in hundred thousands should be included in the caption.

When multiple data sets are graphed together, a legend is generally needed on the graph. **Too much** information on a single graph can be messy and confusing. A good rule of thumb is to include between four and six data sets on any given graph. If there is a good reason to show many data sets together, a second set of graphs (with smaller collections of data) may be included.

If a single data set and a regression line are shown, then the caption might indicate the difference—such as that the regression line is dashed. When choosing colors or patterns for data sets on a graph, you should consider whether the figures might be shown in black-and-white (in which case patterns should be used). Patterns for lines or curves might include solid, dashed, dot-dashed, or thick. Patterns for a bar graph, pie chart, or histogram might include vertical lines, sparse dots, crosshatches, etc.

You should take care to ensure a consistent style when creating figures. For instance, if none of your graphs has a title, a caption should be included for each one to describe it. Alternatively, if each of your graphs has a title, few captions (or none at all) should be used. Generally, it is easiest to leave any numbering to be done in the typesetting software (LaTex or Word, for instance) rather than to try to number figures in the source (Excel or Mathematica, for instance).

Tips for Formatting and Drafting Papers in Mathematics

When creating any kind of bulleted list with numeric entries, avoid using dashes; the number will then appear to be negative. Avoid using open circles that may appear as zeros.

Use consistent formatting with bulleted lists; probably omit periods.

Be particularly careful as you copy information across software or platforms. Copying from one software source, such as Mathematica, into a typesetting product, such as a Microsoft product (Word or PowerPoint, in particular) or Google doc, often takes extra care. In Mathematica, specifically, selecting a code or an image and saving it as a .pdf is often the easiest way. Closing a Mathematica file and reopening but not executing prior to copying will eliminate the unnecessary labels of In/Out. Switching platforms, such as from a Mac to a Windows-based operating system, can cause alarming and unfortunate alterations. At this time, Google Docs generally has fewer formatting capabilities than other typesetting environments, but it has the advantages of portability, ease of communal composing and editing, and recoverability of previous versions.

Try these two tricks to control graphics you've pasted into Word: maintain blank lines before and after an image, and paste your images inside a Word table. (For the latter, make several "paragraphs" with the return key; on each line, type space-tab-space-tab; select the lines and ConvertTextToTable; then paste within a cell of the table).

In a typesetting environment such as LaTex, remember that an image file must be saved in the same folder as the typesetting file. For anyone expecting to continue their studies in the mathematical or computational sciences, learning LaTex is a valuable skill. Using Microsoft Office (Powerpoint, Excel, Word) can result in unintentional and irreversible formatting changes as information is transferred between programs.

General Tips for Writing and Editing in Mathematics

When writing your paper, especially the first draft, it might be helpful to **focus on the wording first and leave the insertions of figures for later.** Include a note, such as [Insert graph of Measles in China here] to mark the space for the table or graph. The same is true with drafting presentation slides. This helps to maintain the flow of the writing process.

In the editing process, having multiple versions in separately named and maintained files is advisable. Back up files on independent media (Google Drive, campus server, USB-drive, external hard drive, CD). A good practice is to save a file with course_LastName_version, such as "Discrete_Knuth_V2a". Many times, an author will modify a description, only to wish to return to the previous wording. (Here, Google Docs has an advantage.) The longer it would take to reproduce the work if lost, the more precautions should be taken in saving all related files.

When creating slides, be careful about applying backgrounds or using a lot of different colors. **Keeping slideshows simple will prevent file corruption (as you move between devices/programs) and reader confusion.** If the content is good, the layout is of little importance; if the content is poor, a snazzy set of slides will not compensate. For a slide presentation, be sure to be prepared with the file saved on a portable drive and your SU GoogleDrive, at least. Save it in multiple formats, including .pdf.

Further Resources for Writing in Mathematics

- Bellare, Mihir. "Technical Writing for Computer Scientists." Department of Computer Science & Engineering, University of California at San Diego. (n.d.)
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- "Writing in Mathematics, Statistics and Computer Science Courses. A Sampling of Advice from Faculty." Writing Across the Curriculum of Marquette University. Marquette University, 2014.

Works Cited:

- <u>CUPM Curriculum Guide 2004</u>, Committee on the Undergraduate Program in Mathematics, Mathematical Association of America, 2004.
- Krantz, Steven G. A Primer of Mathematical Writing. American Mathematical Society, 1997.